A theory of learning dynamics in perceptual decision-making

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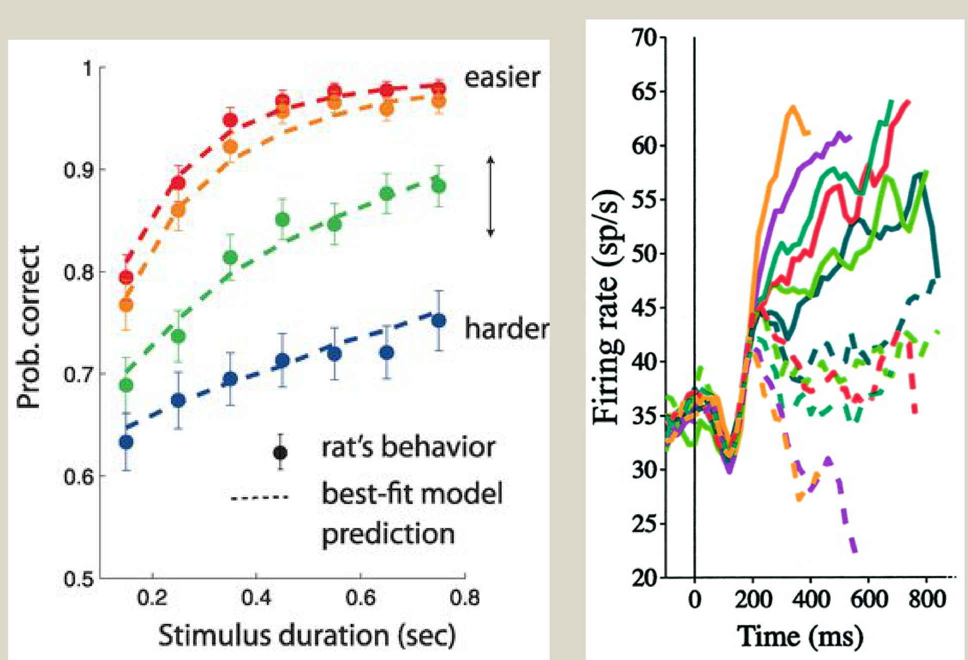
Accumulating decision evidence

- Humans and animals near-optimally accumulate information to make decisions
- Neural correlates of accumulation have been found in areas including LIP and FOF, and predict certainty (Kiani & Shadlen 2009)



Brunton, Botvinick, & Brody 2013 Roitman & Shadlen 2002

- Hand-designed neural network models can reproduce this behavior (Usher & McClelland 2001, Wong & Wang 2006, Bogacz et al. 2006), but do not include the learning process
- **We derive an analytically tractable model of the entire learning trajectory**

Questions

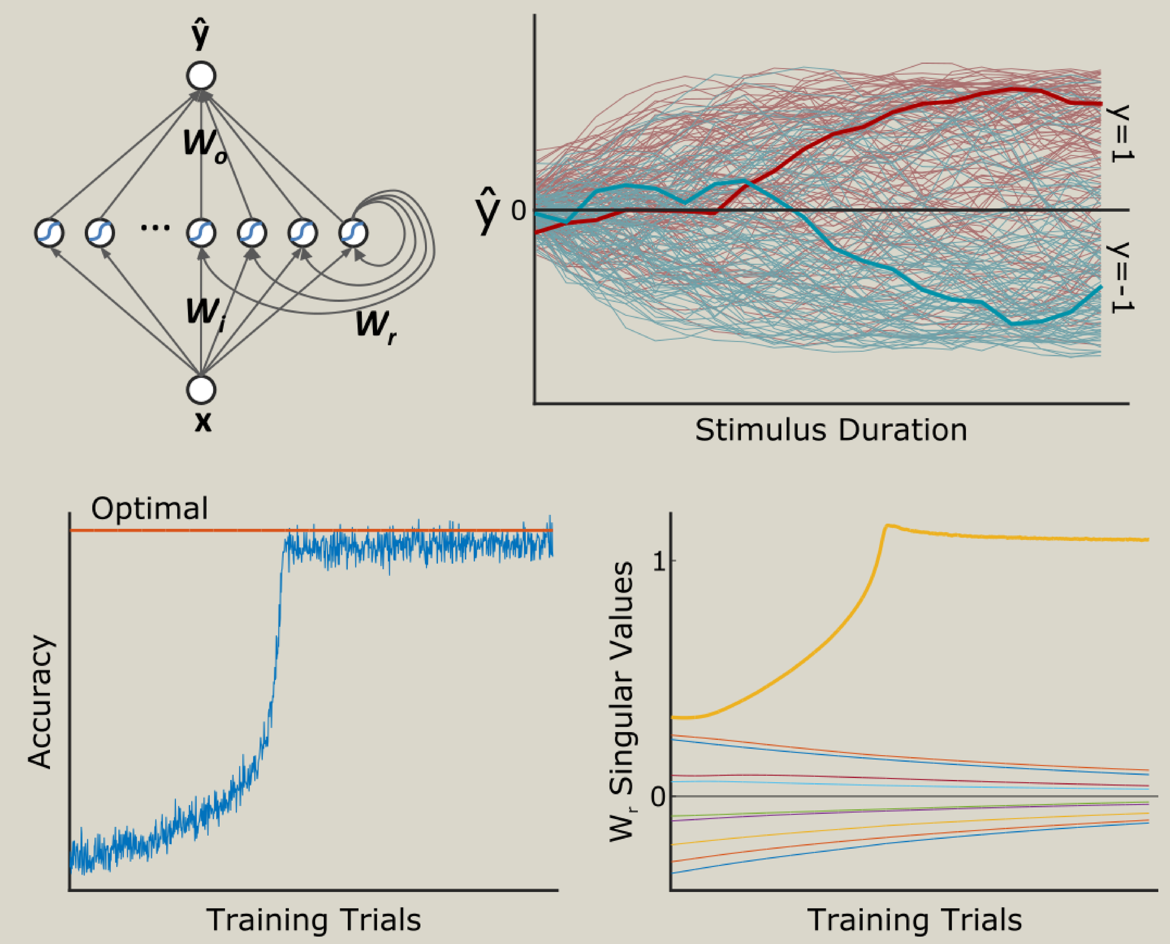
- Does integrating information require specially-designed neural hardware?
- Can we capture nonlinear learning dynamics with a simple model?
- Can we describe learning trajectories in closed form?
- Can we predict experimental data?

Assumptions and notation

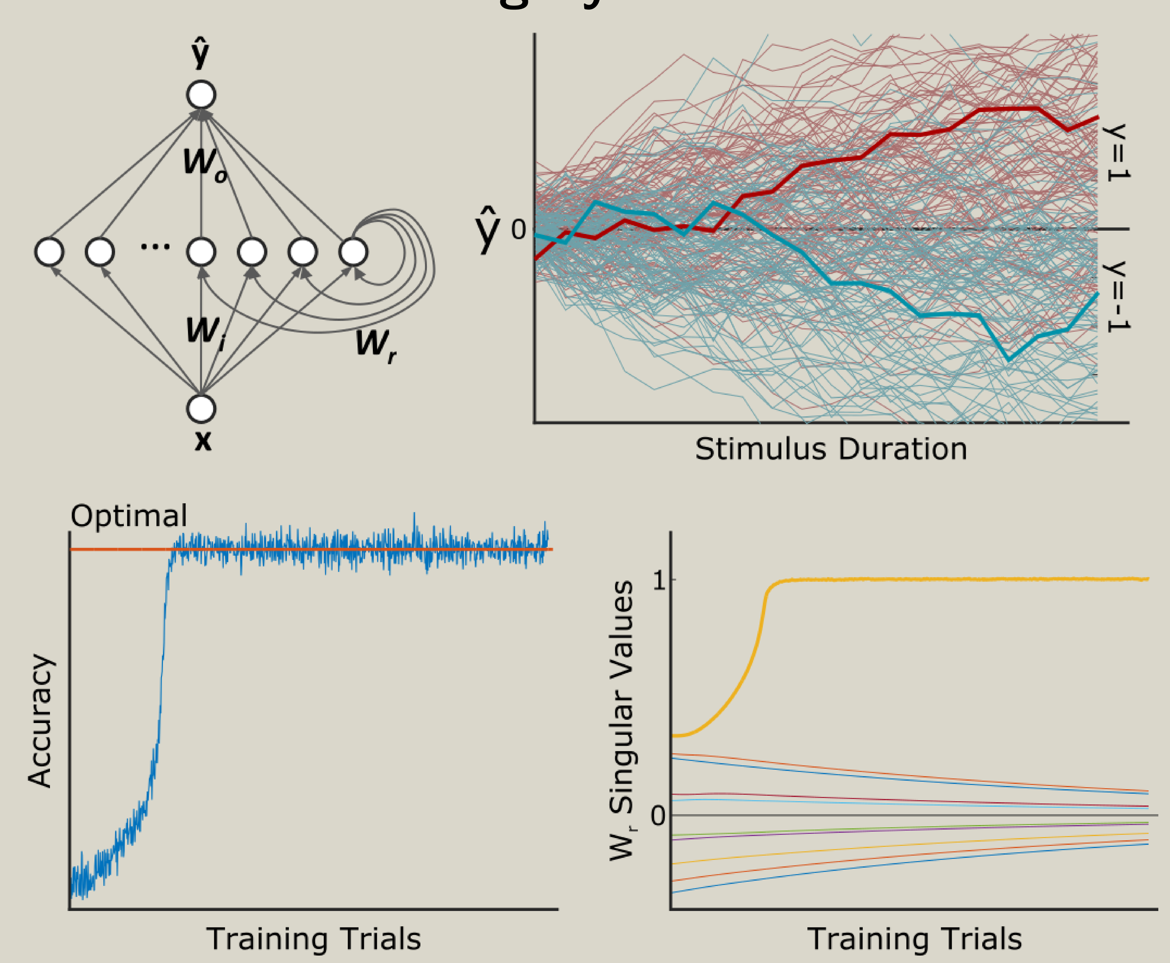
- True stimulus label $y \in \{-1, 1\}$
- Stimulus gives us information at a rate A and with noise c^2: $x_t \sim N(Ay\,dt, c^2\,dt)$

Neural network models

A generic deep recurrent network can learn to make optimal decisions



A linear deep network captures nonlinear learning dynamics



Gradient descent still nonconvex and coupled, e.g.

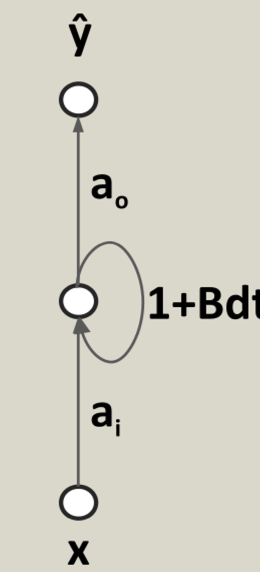
$$\Delta W_o \propto \left[\left(\sum_{t=1}^{D} \Sigma^{yx_t} {W_i}^T ({W_r}^{D-t})^T\right) - W_o \sum_{t_1=1}^{D}\sum_{t_2=1}^{D} {W_r}^{(D-t_1)} W_i \Sigma^{x_{t_1}x_{t_2}} {W_i}^T ({W_r}^{(D-t_2)})^T\right]$$

where Σ^{yx_t} is the input-output cross correlation and $\Sigma^{x_{t_1}x_{t_2}}$ is the input cross correlation

Continuous-time scalar modes

Output dominated by the largest singular value of W_r:

$$\hat{y} = \sum_{t=1}^{T} W_o W_r^{T-t} W_i x_t = \sum_{t=1}^{T} a_o (1 + B\,dt)^{T-t} a_i x_t$$



Taking a continuous-time limit $dt \to 0$:

$$\mathrm{E}[\hat{y}] = \frac{A a_i a_o y (e^{BT} - 1)}{B} \qquad \mathrm{Var}[\hat{y}] = \frac{c^2 a_i^2 a_o^2 (e^{2BT} - 1)}{2B}$$

Output from continuous-time scalar mode is an Ornstein-Uhlenbeck process

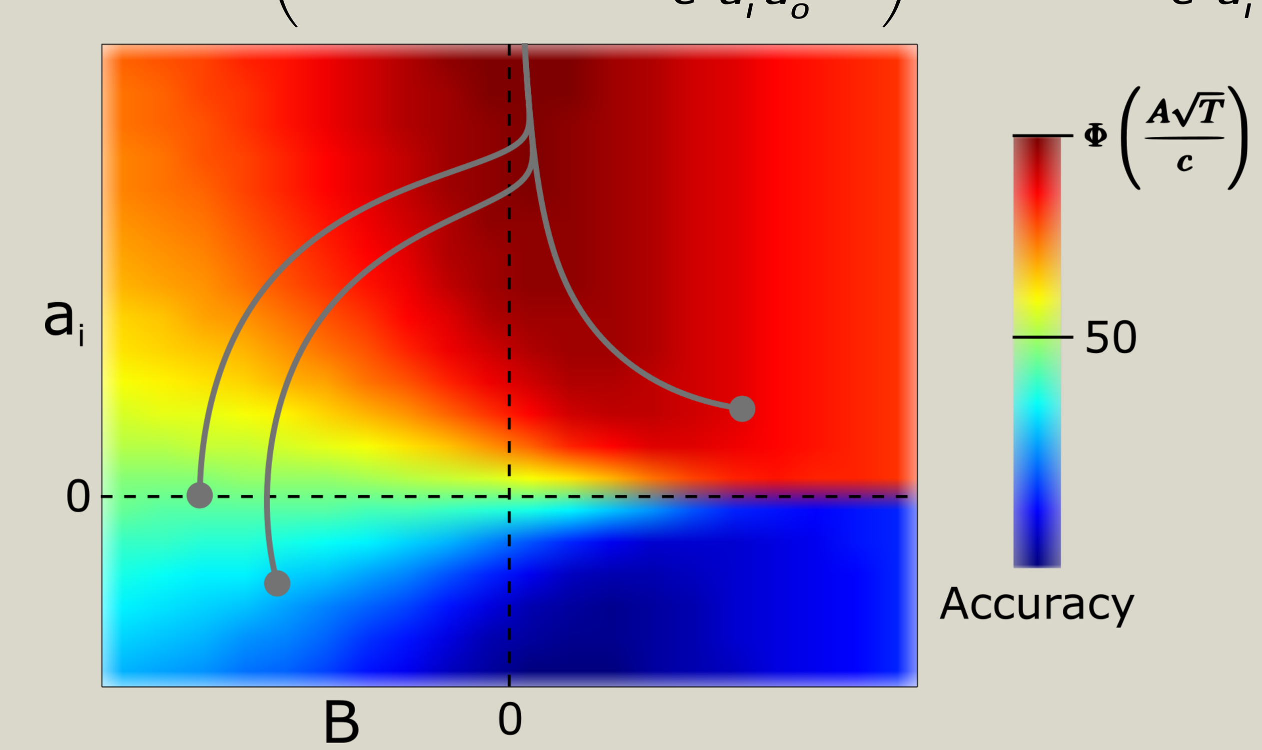
$$d\hat{y} = (B\hat{y} + A)\,dt + c\,dW$$

Gradient descent dynamics

$$\text{Accuracy} = \Phi\left(\frac{A a_i a_o (e^{BT} - 1)}{B\sqrt{c^2 a_i^2 a_o^2 (e^{2BT} - 1)/(2B) + 1}}\right)$$

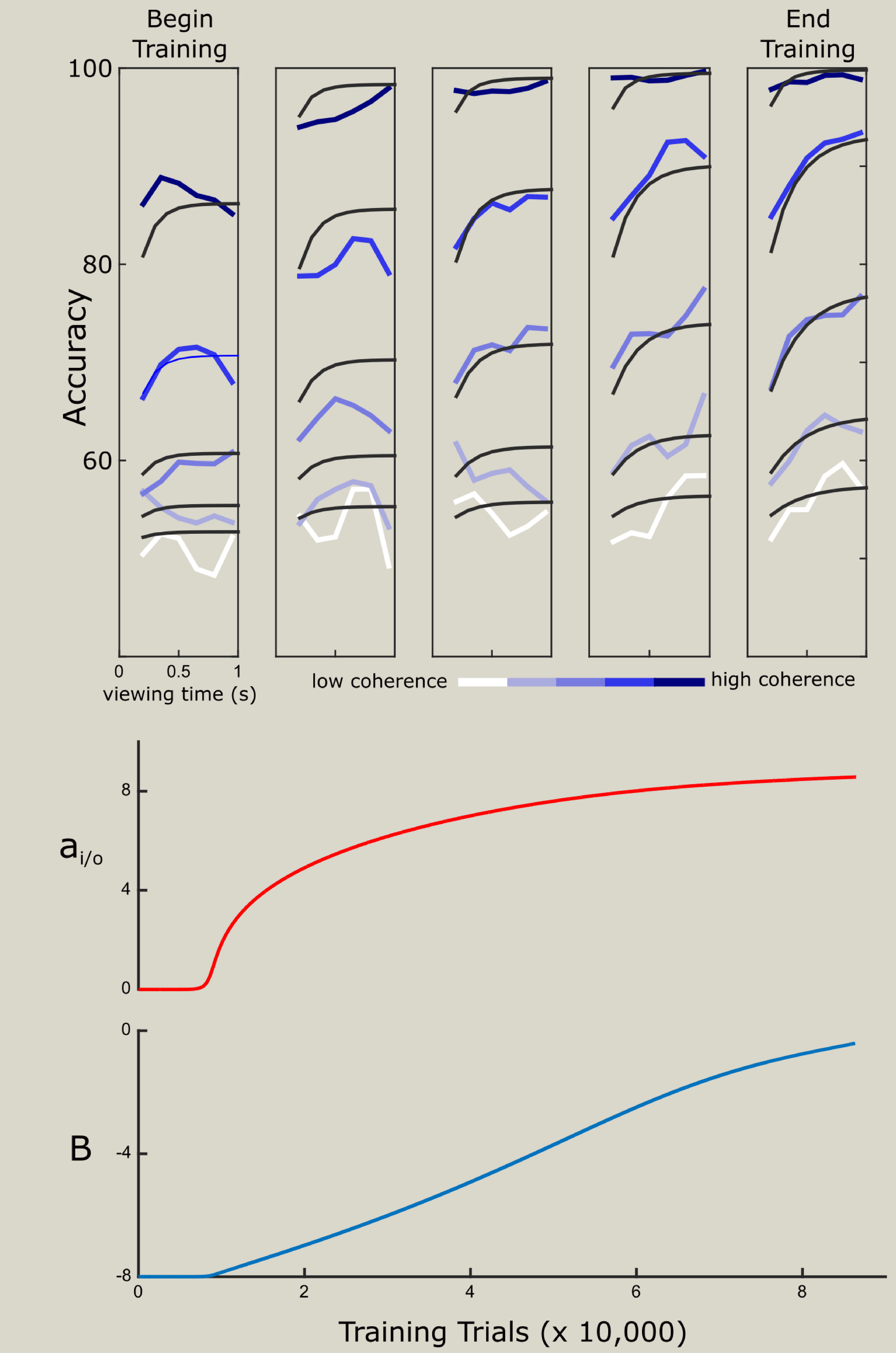
$$\tau \frac{d}{dt} a_{i/o} = e^{\frac{A^2 a_i^2 a_o^2 (e^{BT}-1)}{Bc^2 a_i^2 a_o^2 (e^{2BT}-1) + 2B^2}} \frac{A a_{i/o} (e^{BT} - 1)/B}{\sqrt{2\pi}(a_i^2 a_o^2 c^2 (e^{2BT} - 1)/(2B) + 1)^{3/2}}$$

$$\tau \frac{d}{dt} B = e^{\frac{A^2 a_i^2 a_o^2 (e^{BT}-1)}{Bc^2 a_i^2 a_o^2 (e^{2BT}-1) + 2B^2}} \frac{A a_i^3 a_o^3 c^2}{4\sqrt{2\pi} B^3 (a_i^2 a_o^2 c^2 (e^{2BT} - 1)/(2B) + 1)^{3/2}} \left(-e^{3BT} + (2BT + 1)e^{2BT} + \left(-2BT + 1 + \frac{4B(BT - 1)}{c^2 a_i^2 a_o^2}\right) e^{BT} - 1 + \frac{4B}{c^2 a_i^2 a_o^2}\right)$$



Preliminary fit to experimental data

Model dynamics predict changes in stimulus sensitivity and integration performance



Monkey behavioral data from a random-dot motion task (Law & Gold 2008)

Model features

- Extends drift diffusion model to include learning dynamics
- Links phenomenological model to learning in neural population
- Exact solutions, allowing exact log likelihood computation
- Predicts learning-driven changes for both behavioral and neural data